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TURBULENT BOUNDARY LAYER OF AN INCOMPRESSIBLE FLUID ON A POROUS WALL

By L. E. Kalikhman

Translation

Turbulentnyi pogranichnyi sloi neszhimaemoi zhidkosti na poristoi stenke. Zhurnal Tekh. Fiziki, vol. 25, no. 11, 1955, pp. 1957-1964.

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ABSTRACT

An approximate solution is given of the problem of the turbulent boundary layer of an incompressible flow on a porous wall. The thermal flows that must be removed from the wall for maintaining its given level of temperature and the temperature of the heat insulated porous surface are considerably lowered with the increase in the intensity of the blowing, particularly in utilizing the heat of vaporization. The tests satisfactorily confirm the proposed simple theoretical formulas for computing the resistances and heat transfers of a porous wall.

INDEX HEADINGS

Flow, Turbulent 1.1.3.2

Boundary Layer, Internal Aerodynamics 1.4.7

TURBULENT BOUNDARY LAYER OF AN INCOMPRESSIBLE FLUID ON A POROUS WALL*

By L. E. Kalikhman

One of the effective methods of controlling the boundary layer is through the suction or blowing of a liquid (gas) in the boundary layer. This means may be taken, for example, with the object of decreasing the surface friction, preventing separation of the flow, reducing the heat transfer, or protecting the surface from high temperature.

The existing theoretical studies referring predominantly to the laminar boundary layer on a porous wall are fundamentally systematized in the monograph of Schlichting (ref. 1).

The inflow of fluid through a wall indicates the presence of a normal component of the velocity on the inner boundary of the boundary layer and a decrease of the stability of the laminar sublayer near the wall. Both these circumstances approximate the conditions in the boundary layer on a porous wall to the conditions in the boundary layer of a free stream.

We choose the x and y axes along and normal to the plate, respectively; the origin of coordinates is on the leading edge of the plate. The differential equations of the two-dimensional steady turbulent boundary layer of an incompressible fluid then have the form

$$\rho\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=\frac{\partial \tau}{\partial y} \tag{1}$$

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{2}$$

$$\rho g c_p\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=\frac{\partial q}{\partial y} \tag{3}$$

where

$$\tau=\rho l^2\left(\frac{\partial u}{\partial y}\right)^2 \qquad q=\rho g c_p l^2\,\frac{\partial u}{\partial y}\,\frac{\partial T}{\partial y} \tag{4}$$

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*Turbulentnyi pogranichnyi sloi neszhimaemoi zhidkosti na poristoi stenke. Zhurnal Tekh. Fiziki, vol. 25, no. 11, 1955, pp. 1957-1964.

are the friction stress and the unit heat flow, respectively; u and v are the projections of the velocity on the x and y axes, respectively; ρ is the density of the flow; T is the temperature; c_p is the heat capacity of unit weight; and l is the mixing length, which we shall assume constant in a cross section.

The boundary conditions of the problem are

$$\left.\begin{array}{lllll} \text{for} & y = 0 & u = 0 & v = v_0 & T = T_w \\ \text{for} & y = \delta & u = U & \dfrac{\partial u}{\partial y} = 0 & T = T_b \end{array}\right\} \tag{5}$$

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where U and T_b are the velocity and temperature of the flow outside the boundary layer, respectively, and δ is the thickness of the layer.

By introducing the stream function ψ by the equations

$$u = \frac{\partial\psi}{\partial y} \qquad v = -\frac{\partial\psi}{\partial x}$$

we obtain

$$\frac{\partial\psi}{\partial y}\frac{\partial^2\psi}{\partial x\,\partial y} - \frac{\partial\psi}{\partial x}\frac{\partial^2\psi}{\partial y^2} = 2l^2\frac{\partial^2\psi}{\partial y^2}\frac{\partial^3\psi}{\partial y^3} \tag{6}$$

By assuming that the velocity profiles in the different sections of the boundary layer are similar, we set

$$\psi = U\delta f(\eta) \qquad \eta = \frac{y}{\delta} \tag{7}$$

Substituting the values of the derivatives of the function ψ in equation (1) and separating the variables give

$$-\frac{1}{f}\frac{d^3f}{d\eta^3} = \frac{\delta^2}{2l^2}\frac{d\delta}{dx} \tag{8}$$

The left side of equation (7) depends only on η, the right side only on x; hence, they are each equal to a constant magnitude which we denote by m^3.

We obtain the two equations

$$\frac{d^3f}{d\eta^3} + m^3f = 0 \tag{9}$$

$$\frac{\delta^2}{2l^2}\frac{d\delta}{dx} = m^3 \tag{10}$$

The solution of equation (9) has the form

$$f(\eta) = C_1 e^{-m\eta} + C_2 e^{\frac{m\eta}{2}} \cos \frac{\sqrt{3}}{2} m\eta + C_3 e^{\frac{m\eta}{2}} \sin \frac{\sqrt{3}}{2} m\eta \tag{11}$$

The boundary conditions (4) give

$$\left.\begin{aligned} f'(0) &= 0 \qquad f(0) = -\frac{v_0}{U\delta'} = \xi \\ f' - (1) &= 1 \qquad f''(1) = 0 \end{aligned}\right\} \tag{12}$$

We solve the problem for the case where the parameter ξ maintains a constant value over the length of the plate. From the four conditions in equations (12) we obtain

$$\left.\begin{aligned}
C_1 &= \frac{-\frac{1}{m} e^{-\frac{m}{2}} - \frac{2\xi}{\sqrt{3}} \sin \frac{\sqrt{3}}{2} m}{e^{-\frac{3}{2} m} - \cos \frac{\sqrt{3}}{2} m - \sqrt{3} \sin \frac{\sqrt{3}}{2} m} \\
C_2 &= \frac{\frac{1}{m} e^{-\frac{m}{2}} + \xi\left(e^{-\frac{3}{2} m} - \cos \frac{\sqrt{3}}{2} m - \frac{1}{\sqrt{3}} \sin \frac{\sqrt{3}}{2} m\right)}{e^{-\frac{3}{2} m} - \cos \frac{\sqrt{3}}{2} m - \sqrt{3} \sin \frac{\sqrt{3}}{2} m} \\
C_3 &= \frac{-\frac{\sqrt{3}}{2} e^{-\frac{m}{2}} + \frac{\xi}{\sqrt{3}}\left(-e^{-\frac{3}{2} m} + \cos \frac{\sqrt{3}}{2} m - \sqrt{3} \sin \frac{\sqrt{3}}{2} m\right)}{e^{-\frac{3}{2} m} - \cos \frac{\sqrt{3}}{2} m - \sqrt{3} \sin \frac{\sqrt{3}}{2} m}
\end{aligned}\right\} \tag{13}$$

$$\xi = \frac{\frac{1}{m} e^{-\frac{3}{2} m} + \frac{2}{m} \cos \frac{\sqrt{3}}{2} m}{e^{\frac{m}{2}} - e^{-m} \cos \frac{\sqrt{3}}{2} m - \sqrt{3} e^{-m} \sin \frac{\sqrt{3}}{2} m} \tag{14}$$

The coefficient of friction will be

$$c_p = \frac{2\tau_w}{\rho U^2} = 2\left(\frac{l}{\delta}\right)^2 f''^2(0) \tag{15}$$

where

$$f''(0) = m^2 \frac{-\frac{3}{m} e^{-\frac{m}{2}} + \xi\left(-e^{-\frac{3}{2}m} + \cos\frac{\sqrt{3}}{2}m - \sqrt{3}\sin\frac{\sqrt{3}}{2}m\right)}{e^{-\frac{3}{2}m} - \cos\frac{\sqrt{3}}{2}m - \sqrt{3}\sin\frac{\sqrt{3}}{2}m} \tag{16}$$

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In particular for the plate without blowing ($v_0 = 0$) we obtain $C_1 = 0.13091$, $C_2 = -0.13091$, $C_3 = 0.22673$, $m = 1.8498$, $f''(0) = 1.344$.

As is known, in a turbulent layer for $Re < 10^7$

$$c_f = 0.0592\ Re^{-0.2} \left(Re = \frac{Ux\rho}{\mu}\right) \tag{17}$$

From equations (15) and (17) we obtain the nondimensional mixing length l/δ for $v_0 = 0$ and assume it to be independent of the blowing

$$\frac{l}{\delta} = b\ Re^{-0.1}\ (b = 0.129) \tag{18}$$

From equations (10) and (18) we have

$$\frac{d\delta}{dx} = 2b^2m^3Re^{-0.2} \tag{19}$$

Integrating equation (19) by making use of the initial condition $\delta = 0$ for $x = 0$ gives

$$\delta = 2.5\ b^2m^3Re^{-0.2}x \tag{20}$$

From the definition of ξ, if account is taken of equation (19), this relation follows:

$$\frac{v_0}{U} Re^{0.2} = -2b^2m^3\xi \tag{21}$$

Thus, the condition of the constancy of ξ along the plate is equivalent to the condition $\frac{v_0}{U} Re^{0.2} = \text{const.}$, that is, the blowing velocity varies in our solution as $Re^{-0.2}$.

The parameter ξ expresses the ratio of the amount of fluid blown through a plate of length x to the amount of the original fluid flowing with velocity U through a section of the boundary layer:

$$\xi = -\frac{\int_0^x v_0 dx}{U\delta} \tag{22}$$

The combination $\frac{v_0}{U} Re^{0.2}$ is the fundamental parameter characterizing the intensity of the boundary-layer blowing. The dependence of the original discharge ξ on the intensity of the blowing is represented in figure 1. The change of thickness of the boundary layer δ as a function of $\frac{v_0}{U} Re^{0.2}$ is shown in figure 2.

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The velocity profile will be

$$\frac{u}{U} = \frac{df}{d\eta}\left(\eta; \frac{v_0}{U} Re^{0.2}\right) \tag{23}$$

The following magnitude was chosen as an independent variable in the construction of the velocity profiles (fig. 3):

$$\frac{y}{x} Re^{0.2} = 2.5\ b^2 m^3 \eta$$

The local and mean coefficient of friction on the basis of equations (15) and (18) will be

$$c_f = 2b^2 f''^2(0) Re^{-0.2} \tag{24}$$

$$C_f = \frac{1}{x}\int_0^x c_f dx = 2.5\ b^2 f''^2(0) Re^{-0.2} \tag{25}$$

Figure 4 shows the dependence on $\frac{v_0}{U} Re^{0.2}$ of the ratio of the coefficient of friction with blowing to its value without blowing. The reduction of the friction surface with blowing is explained by the increase in the thickness of the boundary layer and the corresponding decrease in the velocity gradient at the wall. The results of the computation of the dynamic magnitudes are collected in the table.

From equations (1), (3), and (4) it follows that the velocity profiles and temperature drops are similar:

$$\frac{T - T_w}{T_b - T_w} = \frac{u}{U} \tag{26}$$

Differentiating equation (26) with respect to y and setting $Pr = \mu g c_p/\lambda = 1$ give

$$q_w = \frac{g c_p (T_b - T_w)}{U} \tau_w \tag{27}$$

whence

$$Nu = b^2 f''^2(0) Re^{0.8} \tag{28}$$

$$Nu_m = 1.25\ b^2 f''^2(0) Re^{0.8} \tag{29}$$

where

$$Nu = \frac{q_w x}{(T_b - T_w)\lambda} \qquad Nu_m = \frac{1}{(T_b - T_w)\lambda} \int_0^x q_w dx$$

Hence (fig. 4),

$$\frac{Nu}{Nu_{v_0=0}} = \frac{c_f}{c_{f_{v_0=0}}} = \frac{f''^2(0)}{f''^2_{v_0=0}(0)} \tag{30}$$

Let us determine the difference between the total amount of heat passing through unit surface in unit time (i.e., carried away from the hot stream) and the heat in the heating and possible vaporization of the blown fluid. This excess heat which is required to be removed from the walls will be

$$q'_w = q_w - \left[G c_{p_0} (T_0 - T'_0) + r\rho_0 g v_0\right] \tag{31}$$

where $G = \rho_0 v_0 g = \rho'_0 v'_0 g$ is the weight discharge of the blowing fluid; ρ_0 and T_0 are the density and temperature at the exit from the wall, respectively; ρ'_0 and T'_0 are its density and temperature in the reservoir, respectively, that is, at the entrance to the wall; and r and c_{p_0} are the latent heat of vaporization and the capacity per unit weight of the blown fluid, respectively.

By assuming $\rho_0 = \rho$ and $T_0 \approx T_w$ we obtain

$$Nu' Re^{-0.8} = Nu Re^{0.8} - \frac{c_{p0}}{c_p} \left(\frac{v_0}{U} Re^{0.2}\right) \frac{\overline{T}_w - \overline{T}'_0 + \overline{r}}{1 - \overline{T}_w} \tag{32}$$

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where

$$Nu' = \frac{q_w' x}{\lambda(T_b - T_w)} \qquad Nu = \frac{q_w x}{\lambda(T_b - T_w)} = b^2 f''^2(0)$$

$$\overline{T}_w = \frac{T_w}{T_b} \qquad \overline{T}_0' = \frac{T_0'}{T_b} \qquad \overline{r} = \frac{r}{c_{p0} T_b}$$

Figure 5 shows an example of the computation of the excess thermal flows without vaporization for $\overline{r} = 0$, $c_{p0}/c_p = 1$, $\overline{T}_0' = 0.288$, $\overline{T}_w = 0.3$ and 0.5 (curve a) and with vaporization for $\overline{r} = \frac{538.3}{0.48 \cdot 1000} = 112$, $\frac{c_{p0}}{c_p} = \frac{0.48}{0.24} = 2$, $\overline{T}_0' = 0.288$, and $\overline{T}_w = 0.3$ and 0.5 (curve b). It is of interest to remark that in the presence of vaporization very small relative discharges are already sufficient for maintaining a given temperature at the wall. In the case of the thermal insulation of the surface no heat from the wall is conducted away ($q_w' = 0$). The amount of heat carried away from the stream then goes entirely for heating (and possibly vaporizing) the blown fluid.

From equation (32) with $Nu' = 0$ we obtain

$$\overline{T}_w = \frac{b^2 f''^2(0) + \frac{c_{p0}}{c_p}\left(\frac{v_0}{U} Re^{0.2}\right)\left(\overline{T}_0' - \overline{r}\right)}{b^2 f''^2(0) + \frac{c_{p0}}{c_p}\frac{v_0}{U} Re^{0.2}} \tag{33}$$

Figure 6 presents an example of the computation of $\overline{T}_w$ without vaporization (curve a) and with vaporization (curve b) of water for $T_0' = 288$ and $T_b = 1000^\circ$ K. We see that the injection in the boundary layer even without vaporization considerably lowers the temperature of the heat insulated surface.

The temperature of the heat insulated surface and the approximate magnitude of the thermal flows were experimentally determined in reference 2. The tests were conducted by the injection of air in a round tube of diameter $d = 76.2$ millimeters, $T_b = 700^\circ - 1000^\circ$ K, $T_0' = 294^\circ$ K. The Reynolds number of the flow was $Re_d = v_m d\rho/\mu = 25{,}000/215{,}000$, the discharge $G = 0.55 - 0.058$ kg/M^2 sec. There were measured T_b, T_w, T_0', G. In working up the experimental data we assumed that they refer essentially to a part of the tube, that is, we assumed $\delta = d/2$. By assuming arbitrarily several values of $\frac{v_0}{U} Re^{0.2}$ we find for given Re_d successively: the mean velocity in the tube by the formula

$$\frac{u_m}{U} = 2\int_0^1 \frac{u}{U}\left(1 - \frac{y}{\delta}\right) d\,\frac{y}{\delta} \tag{34}$$

the parameter

$$\frac{v_0}{U} R_\delta^{0.25} = (2.5\ b^2 m^3)^{0.25} \frac{v_0}{U} Re^{0.2} \qquad R_\delta = \frac{U\delta\rho}{\mu} = \frac{Re(d)}{2\frac{u_m}{U}} \tag{35}$$

the discharge

$$G = \frac{2g\mu}{d} R_\delta^{0.75} \left(\frac{v_0}{U} R_\delta\right)^{0.25} \tag{36}$$

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Thus, the results of the tests may be represented as a function of $\frac{v_0}{U} Re^{0.2}$ as in figure 6. We see that the agreement of the theoretical curve (without vaporization) with the experimental is entirely satisfactory.[1] The unit flow of heat q_w through the surface for a steady conducting of the test may be determined from the change of the enthalpy of the injected air

$$q_w = Gc_p(T_0 - T_0')$$

The temperature T_0 was not measured in the test. By varying approximately T_0 through T_w, we evidently obtain somewhat higher values of q_w, since actually an underheating of the injected air is possible within the wall with consequent underheating of this air in the mixture with the hot flow. The results of this evaluation of the test data are given in figure 4 which shows that the character of the theoretical dependence is essentially confirmed.

CONCLUSIONS

An approximate solution is given of the problem of the turbulent boundary layer of an incompressible flow on a porous wall. It is shown that the fundamental parameter defining the process is the intensity of the injection $\frac{v_0}{U} Re^{0.2}$ connected with the relative discharge ξ. With increase in $\frac{v_0}{U} Re^{0.2}$ the thickness of the boundary layer increases, the velocity profiles become less full, and the frictional resistances of the plate decrease. The thermal flows that must be removed from the wall for maintaining its given level of temperature and the temperature of the heat

[1]The experimental data were also worked on the assumption that the thickness of the boundary layer at the start of the test portion was equal to zero. The results practically did not change.

insulated porous surface are considerably lowered with the increase in the intensity of the blowing, particularly in utilizing the heat of vaporization. The tests satisfactorily confirm the proposed simple theoretical formulas for computing the resistances and heat transfers of a porous wall.

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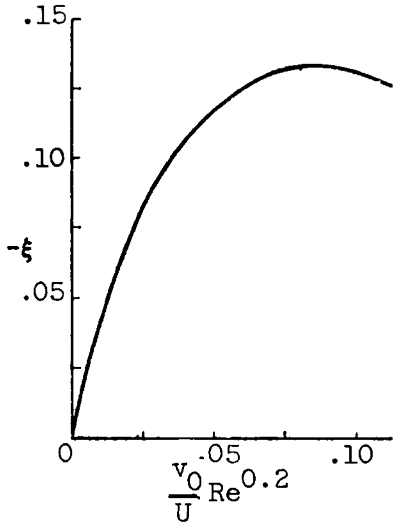

Figure 1.- Dependence of original discharge ξ on intensity of blowing.

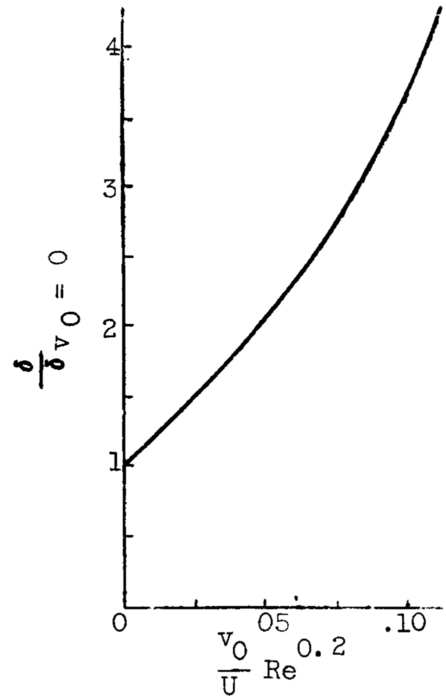

Figure 2.- Change of thickness of boundary layer δ as function of $\frac{v_0}{U}Re^{0.2}$.

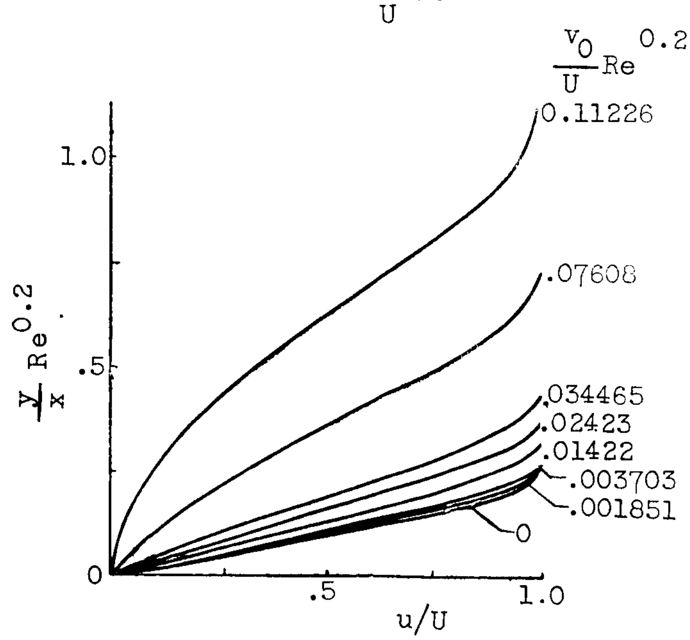

Figure 3.- Velocity profiles for different blowing intensities.

m	ξ	$\frac{v_0}{U}\mathrm{Re}^{0.2}$	$\frac{\delta_{v_0=0}}{\delta}$	$\frac{C_t}{Ct_{v_0=0}}$	C_1	C_2	C_3	$f''(0)$
1.8498	0	0	1	1	0.13091	—0.13091	0.22673	1.34
1.87	—0.0086	0.00185	1.033	0.97	0.1234	—0.13202	0.21871	1.32
1.89	—0.0167	0.00370	1.067	0.94	0.11625	—0.13295	0.21099	1.30
2.0	—0.054	0.0142	1.26	0.79	0.08158	—0.13567	0.17253	1.20
2.1	—0.07964	0.0242	1.46	0.65	0.05571	—0.13535	0.14248	1.09
2.2	—0.098	0.0345	1.68	0.53	0.03451	—0.13303	0.11665	0.98
2.5	—0.128	0.0658	2.47	0.215	—0.00954	—0.11874	0.05754	0.62
2.6	—0.132	0.0761	2.78	0.14	—0.01925	—0.11251	0.04273	0.50
2.7	—0.133	0.086	3.11	0.079	—0.02703	—0.10591	0.02994	0.38
2.8	—0.132	0.095	3.47	0.036	—0.03319	—0.09906	0.01887	0.26
2.9	—0.130	0.104	3.85	0.010	—0.03796	—0.09206	0.009317	0.13
3.0	—0.126	0.1123	4.26	0.00026	—0.04137	—0.085176	0.001402	0.022

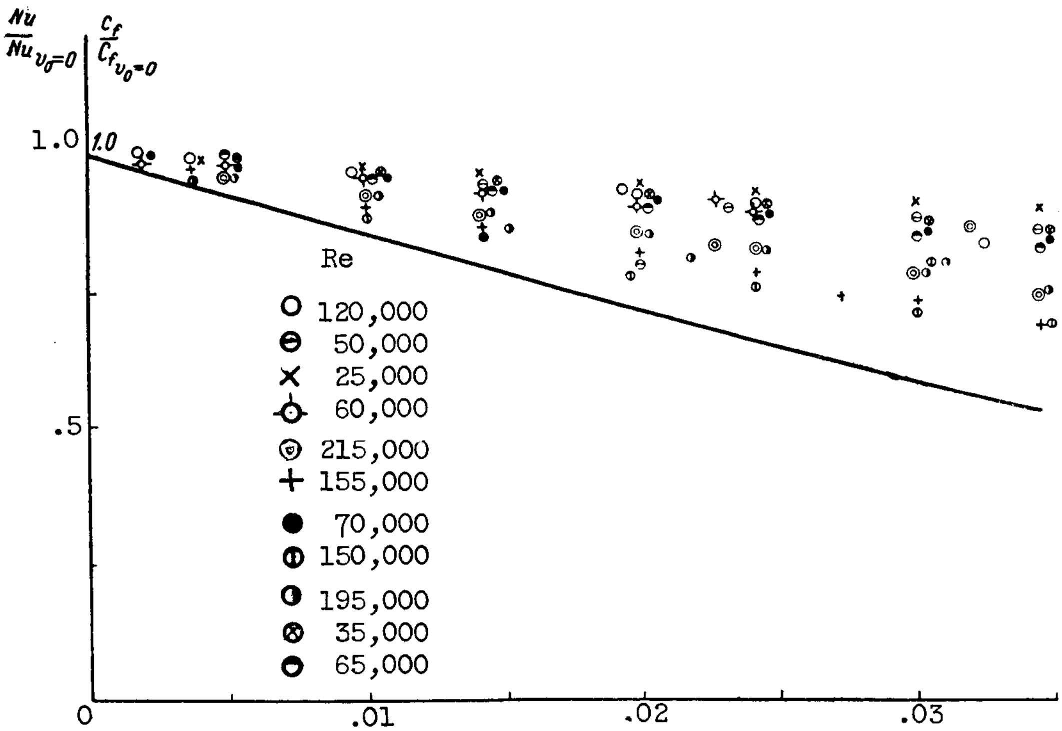

Figure 4. - Resistance and heat transfer of a plane as function of blowing intensity.

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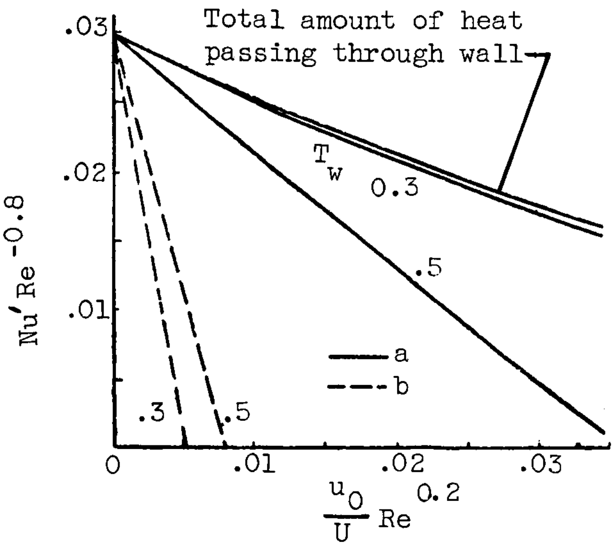

Figure 5. - Excess heat that must be removed from wall.

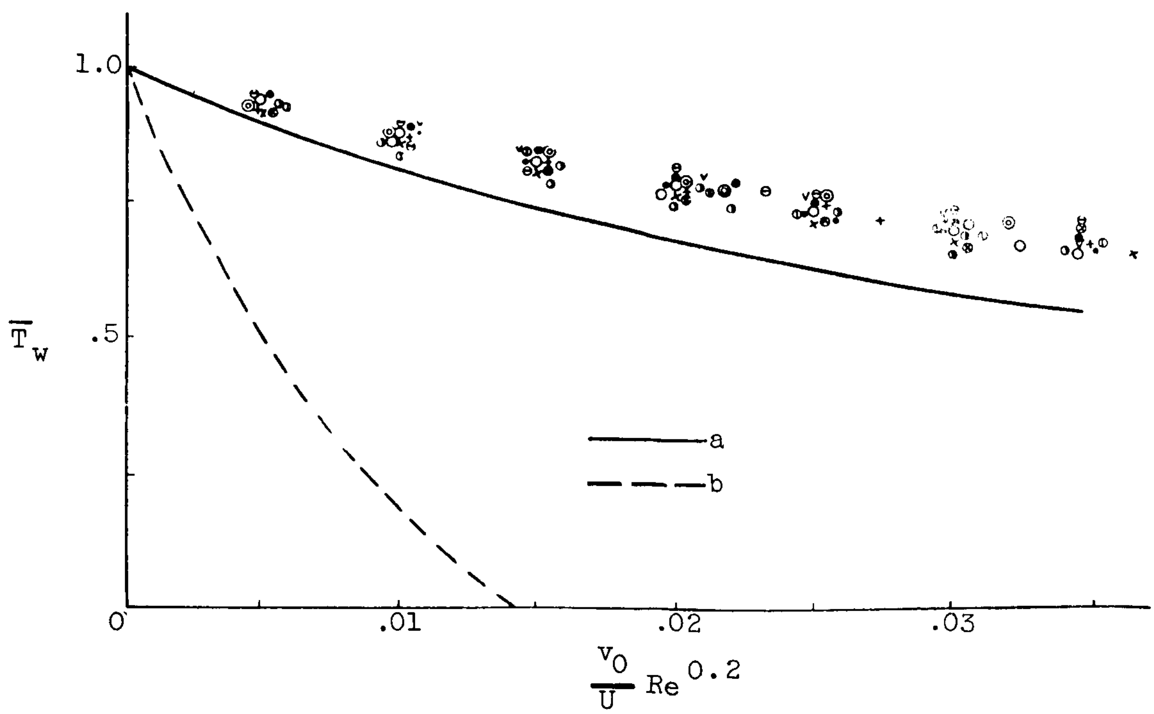

Figure 6. - Relative temperature of surface as function of intensity.